On Algorithms for Asynchronous Track-to-Track Fusion*

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Abstract – *This paper discusses three algorithms for the problem of asynchronous Track-to-Track Fusion (AT2TF) with track delays, where the information configuration of T2TF with partial information feedback (AT2TFpf) is used. This is the most practical configuration for AT2TF with time delays, since communication delays make full information feedback very complicated. The first algorithm is the optimal memoryless AT2TF under the Linear Gaussian (LG) assumption (denoted as AT2TFpfwoMopt), which is also the linear minimum mean square error (LMMSE) fuser without the Gaussian assumption. The second is the Information Matrix Fusion for asynchronous T2TF (denoted as AT2TFpfIMF), which is an extension of the synchronous IMF algorithm. Both algorithms are novel. The third is an approximate AT2TF algorithm proposed by Novoselsky (denoted as AT2TFpfAppr). Among the three algorithms, only the first one theoretically guarantees the consistency of the fuser. The latter two algorithms involve certain degrees of heuristics. Simulation results show that, for the scenarios considered, AT2TFpfIMF is also consistent and has similar level of tracking accuracy as AT2TFpfwoMopt. On the other hand, AT2TFpfAppr has consistency problems. Further more, due to the simplicity of AT2TFpfIMF compared to AT2TFpfwoMopt, it is an appealing candidate for practical applications.*

Keywords: Tracking, Asynchronous Track-to-Track fusion

1 Introduction

Algorithms for synchronous track-to-track fusion (T2TF) have been widely studied. For the optimal T2TF, it is critical to take into account the crosscovariances between tracks of the same target due to common process noises [2] and information feedback [13]. The optimal memoryless T2TF with no information feedback (T2TFwoMnf) was studied in [3, 9]. In [13], the complete set of information configurations for T2TF and the optimal algorithms for the synchronous T2TF were presented, which includes T2TF without memory with no, partial and full information feedback (designated as T2TFwoMnf, T2TFwoMpf and T2TFwoMff, respectively), as well as T2TF with memory with no, partial and full information feedback (designated as T2TFwMnf, T2TFwMpf and T2TFwff). Another type of T2TF algorithm — the information matrix fusion (IMF) — was proposed in [12, 8], and is a special case of track-to-track fusion with memory (T2TFwM). A significant advantage of IMF over the optimal T2TF is that it does not require the crosscovariances between the local tracks, which greatly simplifies its implementation. However, IMF is optimal only when the fuser is operating at full rate [8, 7]. For reduced rate, IMF is heuristic. As reported in [6], IMF has consistency problems for extremely large process noise levels, while for most tracking scenarios it is consistent and has good tracking accuracy.

In the real world, synchronization cannot be achieved and local trackers usually work asynchronously, with local measurements obtained and local tracks updated at different time instants. In addition, communication between local trackers and the fusion center (FC) are subject to possible delays, thus the fusion of delayed tracks should also be addressed. For distributed tracking systems with communication delays, the most practical information configuration is the track-to-track fusion with partial information feedback, where, after each fusion, only one local track assumed collocated with the FC is updated with the fused track (partial information feedback), while other local trackers operate by themselves without information feedback from the FC.

Early attempts to address the asynchronoous T2TF (AT2TF) problem include [5], where the problem is converted and solved as the fusion of out-of-sequence measurements (OOSM). However, the algorithm only

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reconstructs and fuses the information of the latest local measurements. A pseudo measurement approach of fusing asynchronous tracks can be found in [10]. In [11], three approximate algorithms for AT2TF are proposed. Later in the present paper, the algorithm form [11] with the best performance is evaluated by simulations, which show that this algorithm has consistency problems. This is because the crosscorrelation between the central and local tracks is not adequately accounted.

In this paper we generalize the optimal (under linear Gaussian — LG — assumption) synchronous T2TF algorithm from [13] for the AT2TF problem, where the information configuration of memoryless fusion with partial information feedback (believed to be the most practical one) is used. For the fusion, this algorithm accounts exactly the crosscovariances between the local tracks. It handles both the asynchronous sampling times of the local trackers and the fusion of delayed tracks, and guarantees the consistency of the fused estimates. The more complicated version of AT2TF with memory is not considered here, due to the limited gain in tracking accuracy, especially when significant geometric diversity exists among the local tracks[1].

The Information Matrix Fusion (IMF) [12, 6] for synchronous T2TF is theoretically optimal for fusion only at full rate. In this paper, we extend IMF further for the fusion of asynchronous tracks with time delays. This fusion algorithm is used with the information structure of partial information feedback[2] and designated as AT2TFpfIMF. It turns out that AT2TFpfIMF, even though heuristic, is remarkably robust. It shows good consistency over the practical range of process noise levels and has tracking accuracy practically as good as AT2TFpfwoMopt. Furthermore, due to the simplicity of its implementation compared to AT2TFpfwoMopt, AT2TFpfIMF is an appealing candidate for practical applications.

The paper is organized as follows. Section 2 formulates the AT2TF problem. Section 3 presents the algorithm AT2TFpfwoMopt. Section 4 generalizes the IMF for AT2TF with delayed tracks, which leads to the algorithm AT2TFpfIMF. Section 5 compares the two algorithms and the approximate algorithm in [11] by sublimations. Conclusions are presented in section 6. For the convenience of readers, Table 1 lists the acronyms used in this paper.

Table 1: List of Acronyms

T2TF	Track-to-Track Fusion
AT2TF	Asnynchronous Track-to-Track Fusion
AT2TFpf	AT2TF with partial information feedback
AT2TFpfwoM	AT2TFpf with no Memory
AT2TFpfwoMopt	The Optimal Algorithm for AT2TFpfwoM
IMF	Information Matrix Fusion
AT2TFpfIMF	The generalized IMF algorithm for AT2TFpf
AT2TFpfAppr	The Approximate algorithm for AT2TFpf in [11]

2 Problem Formulation – AT2TFpf

For the sake of simplicity, consider the basic scenario of the fusion of two tracks of a target from two local trackers[3]. The trackers operate asynchronously at sampling intervals T_1 and T_2. Tracker 1 is collocated with the FC, whose track is available for fusion with no time delay. Tracker 2 is a remote tracker which sends its track $(\hat{x}_2(t_c|t_c),\ P_2(t_c|t_c))$ to the FC once in a while, where communication time t_c is the time stamp of the local track. The track arrives at the FC with a time delay t_D which may change from time to time. When track 2 is received, the FC fuses track 1 with the delayed track 2 at fusion time t_f (with $t_f \geq t_c + t_D$), which can be written as

$$[\hat{x}_c(t_f|t_f),\ P_c(t_f|t_f)] = \mathbf{f}\left[\hat{x}_1(t_f|t_f),\ P_1(t_f|t_f),\ \hat{x}_2(t_c|t_c),\ P_2(t_c|t_c), \ldots\right] \quad (1)$$

where $(\hat{x}_c(t_f|t_f),\ P_c(t_f|t_f))$ is the fused track. Depending on the fusion algorithm to be used, additional information will be required, which is indicated by the "..." in (1). In view of the partial information feedback[4], after the fusion, track 1 continues with the fused track $(\hat{x}_c(k|k),\ P_c(k|k))$ which has improved accuracy, while local tracker 2 operates by itself unaffected by the fusion.

3 The Optimal Algorithm for AT2TFpf with no Memory – AT2TFpfwoMopt

The objective of the AT2TF is to fused track 1, given by $\hat{x}_1(t_f|t_f)$, $P_1(t_f|t_f)$, with the predicted track 2, given by $\hat{x}_2(t_f|t_c)$, $P_2(t_f|t_c)$. Compared to the synchronous T2TF, there are two additional issues to be handled. The first one is that the sampling rates used

[1]In such cases, track estimates from the local trackers provide complementary perspectives of the target state.

[2]When communication delays exist, T2TF with full information feedback is too complicated to implement even with the heuristic IMF.

[3]The problem of track-to-track association is not considered.

[4]T2TF with full information feedback (T2TFwoMff), is substantially more complicated than T2TFwoMpf due to the transmission delays between the local tracker and the fusion center. In view of the results presented later, it is deemed not practical.

by the two trackers are different. This causes difficulties for the calculation of the crosscovariances between tracks. The solution to this problem is to use the union of the sampling times, where a zero filter gain is used for the track when no update is done for it. Then the crosscovariance between tracks can be calculated similarly to the synchronized case as shown later in (2).

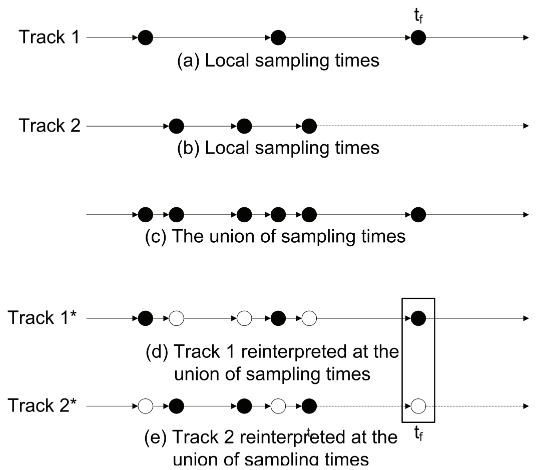

Figure 1: The union of the sampling times

Fig. 1 illustrates the idea of the union of the sampling times. Fig. 1(a) shows the time axis of tracker 1, on which the black circles indicate when tracker 1 received measurements and did actual track updates[5]. Fig 1(b) shows the same for track 2. Fig. 1(c) shows the union of the sampling times at the two trackers on the same time axis. Then tracks 1 and 2 are reinterpreted according to the union of the sampling times in Fig. 1(d)-(e), where the black circles represent actual track updates and the white circles represent virtual track updates, i.e., with zero filter gains.

To differentiate the original tracks and the reinterpreted tracks according to the union of the sampling times, the latter ones are denoted as, $\hat{x}_{i^*}$ with a "*" superscript for the track index. After this, the exact crosscovariance between the two tracks at the any time $t_a > t_l$ is calculated as follows

$$\begin{aligned} P_{1^*2^*}(t_a|t_a) &= W^e_{1^*}(t_a,t_l)P_{1^*2^*}(t_l|t_l)W^e_{2^*}(t_a,t_l)' \\ &+ \sum_{i=l+1}^{f} W^v_{1^*}(t_a,t_{i-1})Q(t_i,t_{i-1})W^v_{2^*}(t_a,t_{i-1})' \end{aligned} \quad (2)$$

where t_l is the most recent time at which the crosscovariance is available (designated as "prior time"); the summation in (2) is over the set $\{t_l,...,t_a\}$, which is the union of the sampling times in the time interval $[t_l\ ,t_a]$; and

$$W^e_{s^*}(t_a,t_l) = \prod_{i=0}^{f-l-1}[I-K_{s^*}(t_{f-i})H_{s^*}(t_{f-i})]F(t_{f-i},t_{f-i-1}) \quad (3)$$

$$\begin{aligned} W^v_{s^*}(t_a,t_{i-1}) &= \\ -&\left\{\prod_{j=0}^{f-i-1}[I-K_{s^*}(t_{f-j})H_{s^*}(t_{f-j})]F(t_{f-j},t_{f-j-1})\right\} \\ \cdot&[I-K_{s^*}(t_i)H_{s^*}(t_i)] \end{aligned} \quad (4)$$

$$s = 1,2$$

where $K_{s^*}(t_i)$, $i = l+1,...,f$ are the local Kalman filter gains, which are zero for the virtual updates; $H_{s^*}(t_i)$ are the observation matrices at local tracker s and $F(t_i,t_{i-1})$ are the state transition matrices from t_{i-1} to t_i. Note that the calculation of the exact crosscovariance between two tracks requires the local filter gains and observation matrices at every sampling time, which puts a high requirement on communication capacity. An approximate approach to save communication cost can be found in [13].

Note that, for the synchronous T2TF, the system can use either the Discretized Continuous-time Kinematic Model or the Direct Discrete-Time Kinematic Model (see [1] Secs. 6.2 and 6.3). However, for AT2TF, the use of the union the sampling times requires to break the local process noises down to finer pieces. To make the tracks consistent before and after the reinterpretation, only the Discretized Continuous-Time Kinematic Model should be used.

The second issue is that the fusion of local estimates with time delays makes it more difficult to calculate the crosscovariance between the local tracks. A scheme is designed for the crosscovariance calculation and the fusion, which is illustrated in Fig. 2. Starting from the first fusion, as shown in Fig. 2(a), t_l denotes the prior time when covariances and crosscovariances between the two tracks, i.e., $P_1(t_l|t_l)$, $P_2(t_l|t_l)$ and $P_{12}(t_l|t_l)$, are available at the FC. Communication time t_c is the time when track 2, namely, $(\hat{x}_2(t_c|t_c),\ P_2(t_c|t_c))$, is sent to the FC. Due to the time delay in data transmission, at fusion time t_f, track $(\hat{x}_1(t_f|t_f),\ P_1(t_f|t_f))$ will be fused with the predicted track $(\hat{x}_2(t_f|t_c),\ P_2(t_f|t_c))$.

The first fusion is done as follows

- Reinterpret both track 1 and the prediction of the delayed track 2 using the union of the sampling times.

- Propagate the prior information from time t_l to t_c

$$P_{1*}(t_c|t_c) = \begin{cases} P_1(t_c|t_c), & \text{if an actual track 1 update happened on } t_c \\ F(t_c,t_{c1})P_1(t_{c1}|t_{c1})F(t_c,t_{c1})' + Q(t_c,t_{c1}), & \text{otherwise} \end{cases} \quad (5)$$

where t_{c1} is the latest time before t_c when track 1 was updated and $Q(t_c,t_{c1})$ is the cumulative effect

[5]It is assumed that the local trackers have no delay between when a measurement is taken and the track update. Delay is assumed in the communication between tracker 2 and the FC.

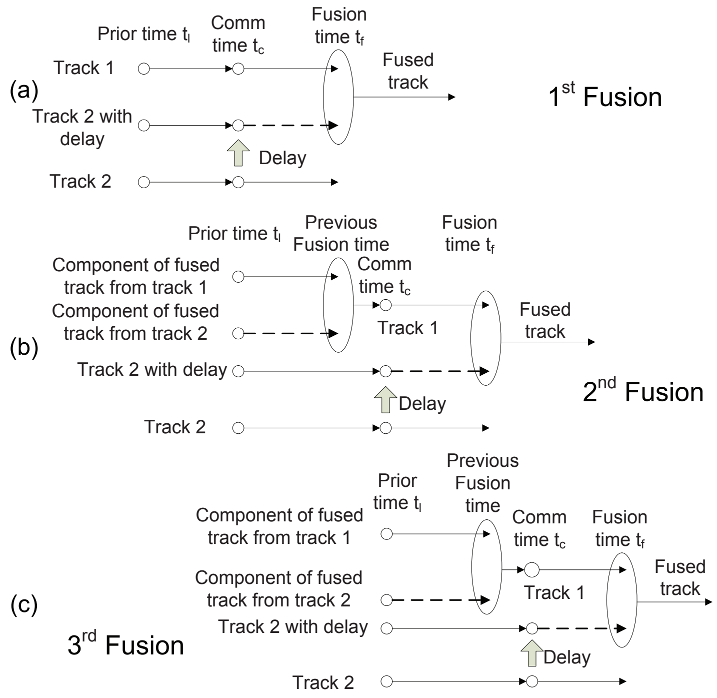

Figure 2: Flowchart of T2TFwoMpf with feedback to tracker 1 and delayed track 2 ("- - - -" shows prediction)

of the process noise in $[t_{c1}, t_c]$.

$$P_{2^*}(t_c|t_c) \;=\; P_2(t_c|t_c) \qquad (6)$$

$P_{1^*2^*}(t_c|t_c)$ can be calculated using (2) from t_l to t_c (7)

- $\hat{x}_{1^*}(t_f|t_f)$ and $P_{1^*}(t_f|t_f)$ are available at the FC.
- With $\hat{x}_2(t_c|t_c)$ and $P_2(t_c|t_c)$ sent to the FC, using prediction it follows that
- Note that, at this point the covariances and crosscovariances between the two tracks, namely, $P_{1^*}(t_c|t_c)$, $P_{2^*}(t_c|t_c)$ and $P_{1^*2^*}(t_c|t_c)$, are available at t_c, which makes t_c the new prior time t_l for the next fusion.

$$\hat{x}_2(t_f|t_c) \;=\; \hat{x}_{2^*}(t_f|t_f) = F(t_f,t_c)\hat{x}_2(t_c|t_c) \qquad (8)$$

$$\begin{aligned} P_2(t_f|t_c) &= P_{2^*}(t_f|t_f) \\ &= F(t_f,t_c)P_2(t_c|t_c)F(t_f,t_c)' + Q(t_f,t_c) \end{aligned} \qquad (9)$$

where $F(t_f,t_c)$ is the state transition matrix from time t_c to t_f and $Q(t_f,t_c)$ is the cumulative effect of the process noises in $[t_c, t_f]$.

- With (5)–(7), the crosscovariance $P_{12}(t_f|t_f;t_f|t_c) = P_{1^*2^*}(t_f|t_f)$ is calculated according to the union of the sampling times using (2) from t_c to t_f.
- With the information above, the optimal AT2TF is done using the LMMSE fuser [1]

$$\begin{aligned} \hat{x}_c(t_f|t_f) \;=\; & \hat{x}_{1^*}(t_f|t_f) + [P_{1^*}(t_f|t_f) - P_{1^*2^*}(t_f|t_f)] \\ & \cdot [P_{1^*}(t_f|t_f) + P_{2^*}(t_f|t_f) - P_{1^*2^*}(t_f|t_f)' - P_{1^*1^*}(t_f|t_f)]^{-1} \\ & \cdot [\hat{x}_{2^*}(t_f|t_f) - \hat{x}_{1^*}(t_f|t_f)] \\ & = \hat{x}_{1^*}(t_f|t_f) + K_{1^*2^*}(t_f)[\hat{x}_{2^*}(t_f|t_f) - \hat{x}_{1^*}(t_f|t_f)] \\ & = (I - K_{1^*2^*}(t_f))\hat{x}_{1^*}(t_f|t_f) + K_{1^*2^*}(t_f)\hat{x}_{2^*}(t_f|t_f) \end{aligned} \qquad (10)$$

$$\begin{aligned} P_c(t_f|t_f) \;=\; & P_{1^*}(t_f|t_f) - [P_{1^*}(t_f|t_f) - P_{1^*2^*}(t_f|t_f)] \\ & \cdot [P_{1^*}(t_f|t_f) + P_{2^*}(t_f|t_f) - P_{1^*2^*}(t_f|t_f) - P_{1^*2^*}(t_f|t_f)']^{-1} \\ & \cdot [P_{1^*}(t_f|t_f) - P_{1^*2^*}(t_f|t_f)'] \end{aligned} \qquad (11)$$

The second fusion, as illustrated in Fig. 2(b), is slightly different from the first fusion in propagating the crosscovariance between track 1 and the delayed track from the prior time t_l to the new communication time t_c. Note that, in Fig. 2, it is assumed that the second communication happens after the previous fusion. For scenarios where this assumption does not hold, the scheme can be easily modified to accommodate the change. The key of the fusion scheme is to propagate the prior information (including covariances and crosscovariances between the tracks) to the latest t_c so that all the local filter gains and fusion gains before t_c can be discarded. After reinterpreting track 1 and the delayed track 2 using the union of the sampling times as in the first step of the first fusion, the crosscovariance between tracks 1 and 2 should be propagated from t_l (which is also the previous communication time) to the current time t_c. Now (2) can not be used directly, since, after the first fusion, track 1 continues with the fused track (partial information feedback), which contains two parts: one from the old track 1 (indicated by index $o1$), the other from the predicted track 2 (indicated by index $o2$). The crosscovariances $P_{1^*2^*}(t_c|t_c)$ for the second fusion is calculated as follows:

- Calculate the crosscovariances $P_{o1^*2^*}(t_{f_p}|t_{f_p})$ and $P_{o2^*2^*}(t_{f_p}|t_{f_p})$ using (2), where t_{f_p} is the previous fusion time.
- From (10)

$$\begin{aligned} P_{1^*2^*}(t_{f_p}|t_{f_p}) &= (I - K_{1^*2^*}(t_{f_p}))P_{o1^*2^*}(t_{f_p}|t_{f_p}) \\ &\quad + K_{1^*2^*}(t_{f_p})P_{o2^*2^*}(t_{f_p}|t_{f_p}) \end{aligned} \quad (12)$$

- Propagate the crosscovariance $P_{1^*2^*}(t_{f_p}|t_{f_p})$ from t_{f_p} to t_c using (2). Now, with the new $P_{1^*2^*}(t_c|t_c)$ calculated, t_c becomes the new prior time for the next fusion and the old prior information can be discarded.

At this point, the rest of the fusion can be done exactly the same as in the first fusion. The third fusion and the ones afterwards follow the same steps as in the second fusion.

4 Generalized Information Matrix Fusion for Asynchronous T2TF with Partial Information Feedback – AT2TFpfIMF

The synchronized version of Information Matrix Fusion (IMF) has been presented in [8]. Operating at full rate, IMF is optimal (equivalent to CMF). At reduced rate, the algorithm is heuristic, but it works remarkably well over the practical range of process noise levels[6]. In this section, we generalize the IMF for asynchronous T2TF and evaluate its performance. Consider the fusion of track 1 at the FC and a delayed local track from tracker 2. At the fusion time t_f, one has $(\hat{x}_1(t_f|t_f),\ P_1(t_f|t_f))$ from track 1 and the delayed track $(\hat{x}_2(t_c|t_c),\ P_2(t_c|t_c))$ from track 2, where $t_c < t_f$. For the information matrix fusion, the fuser also needs the local track from the previous communication time t_{c_p}, which is denoted as $\left(\hat{x}_2(t_{c_p}|t_{c_p}),\ P_2(t_{c_p}|t_{c_p})\right)$. The asynchronous IMF is given as follows:

For the fused covariance, one has

$$\begin{aligned} P_c(t_f|t_f)^{-1} &= P_1(t_f|t_f)^{-1} \\ &\quad + \left(P_2(t_f|t_c)^{-1} - P_2(t_f|t_{c_p})^{-1}\right) \end{aligned} \quad (13)$$

where

$$\begin{aligned} P_2(t_f|t_c) &= F(t_f,t_c)P_2(t_f|t_c)F(t_f,t_c)' \\ &\quad + Q(t_f,t_c) \end{aligned} \quad (14)$$

is the predicted covariance of track 2 from t_c to t_f; and

$$\begin{aligned} P_2(t_f|t_{c_p}) &= F(t_f,t_{c_p})P_2(t_f|t_{c_p})F(t_f,t_{c_p})' \\ &\quad + Q(t_f,t_{c_p}) \end{aligned} \quad (15)$$

[6]Divergence was observed only for extremely large process noise levels. [7]

is the predicted covariance of track 2 from t_{c_p} to t_f. Note that $\left(P_2(t_f|t_c)^{-1} - P_2(t_f|t_{c_p})^{-1}\right)$ in (13) is the net information gain from track 2, which is mainly due to the measurement updates by local tracker 2 from t_{c_p} to t_c, and can be viewed as independent across the local trackers.

For the fused track estimates, one has

$$\begin{aligned} P_c(t_f|t_f)^{-1}\hat{x}_c(t_f|t_f) &= P_1(t_f|t_f)^{-1}\hat{x}_1(t_f|t_f) \\ +(P_2(t_f|t_c)^{-1}\hat{x}_2(t_f|t_c) &- P_2(t_f|t_{c_p})^{-1}\hat{x}_2(t_f|t_{c_p})) \end{aligned} \quad (16)$$

where

$$\hat{x}_2(t_f|t_c) = F(t_f,t_c)\hat{x}_2(t_c|t_c) \quad (17)$$

$$\hat{x}_2(t_f|t_{c_p}) = F(t_f,t_{c_p})\hat{x}_2(t_{c_p}|t_{c_p}) \quad (18)$$

are the predicted track 2 estimates from t_c and t_{c_p} to the fusion time t_f.

After the fusion, only track 1 is updated by the fused track in view of the partial information feedback. Thus

$$\hat{x}_1^*(t_f|t_f) = \hat{x}_c(t_f|t_f) \quad (19)$$

$$P_1^*(t_f|t_f) = P_c(t_f|t_f) \quad (20)$$

Note that this information matrix update does not need the crosscovariances among local tracks, which makes it very simple to implement and directly usable for scenarios with more than two local trackers.

5 Simulation Results

To compare the performance of the previously discussed algorithms for AT2TF using delayed tracks with partial information feedback (from Secs. 2 and 3) and the algorithm AT2TFpfAppr [11], a 2-D tracking scenario is used with two local trackers 1 and 2 tracking one target. The target motion follows a CWNA model in [1] with process noise power spectral density (PSD) $\tilde{q}$.[7] The target state is defined as $x = [\xi\,\dot{\xi}\,\zeta\,\dot{\zeta}]'$, i.e., position and velocity in 2-D Cartesian coordinates, with initial value set, without loss of generality, as $[2000\text{ m}, -2\text{ m/s}, 5000\text{ m}, -5\text{ m/s}]$. Tracker 1 is collocated with the FC at the origin (0,0), while Tracker 2 is located at (X_2, Y_2). Tracker i ($i = 1,\ 2$) takes position measurements of the target in its polar coordinate every T_i with zero mean white noise errors. The range standard deviation is σ_i^r and the azimuth standard deviation is σ_i^a, which are set as 10 m and 1^o, respectively, for both trackers. The local tracks are generated using the Converted Measurement Kalman Filter [1]. Tracker 2 sends its track at prespecified time instants to the FC with a communication delay of T_D. In the following scenarios, all the simulation results are obtained from 100 Monte Carlo runs

[7]As explained in Sec. 3, only the discretized continuous-time kinematic models, e.g., CWNA model, can be used for AT2TFpfwoMopt.

AT2TFpfIMF (Asynchronous Track-to-Track Fusion with Partial Information Feedback using IMF) vs. AT2TFpfwoMopt (The Optimal Asynchronous Track-to-Track Fusion without Memory with Partial Information Feedback)

Scenario 1: Fusion of two tracks with low process noise and significant geometric diversity:
Tracker 2 location: $[5000, 0]$ m
Sampling intervals: $T_1 = 2$ s, $T_2 = 2.5$ s
Process noise PSD: $\tilde{q} = 10^{-1}$ $\mathrm{m}^2/\mathrm{s}^3$
Fusion times: $[11 : 8 : 150]$ s (start from 11 s, every 8 s.)
Communication delay: $T_D = 7$ s (For the fusion, local track 2 is about 7 s behind the fusion time.)

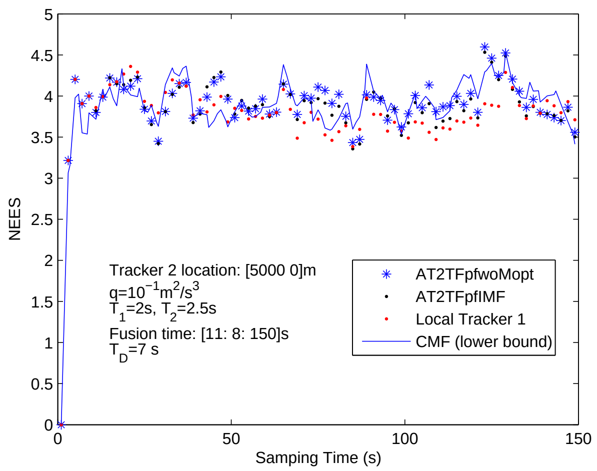

Figure 3: Consistency Test: Scenario 1

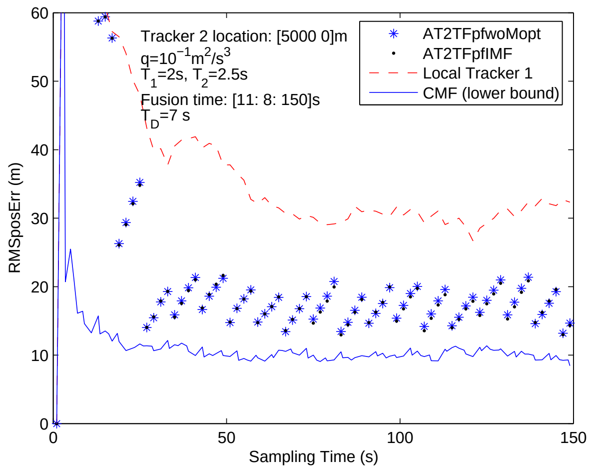

Figure 4: RMS position errors: Scenario 1 (AT2TFpfwoMopt and AT2TFpfIMF are indistinguishable)

Figs. 3–4 compare the performance of AT2TFpfIMF and AT2TFpfwoMopt against the single sensor 1 without fusion and the ideal centralized measurement fusion (CMF) without time delay. It can be seen that AT2TFpfIMF has no consistency problem despite being heuristic. Further simulations show that the consistency holds for a wide range of process noise levels and time delays of the local track. With significant geometric diversity, AT2TFpfIMF and AT2TFpfwoMopt show the same level of fusion accuracy, which is significantly better than that of the single track 1 without fusion. There is a performance gap between these and the ideal CMF which is due to the time delay of track 2.

Scenario 2 Fusion of two tracks with low process noise and little geometric diversity:
Tracker 2 location: $[0, 0]$ m
All other parameters are the same as in scenario 1.

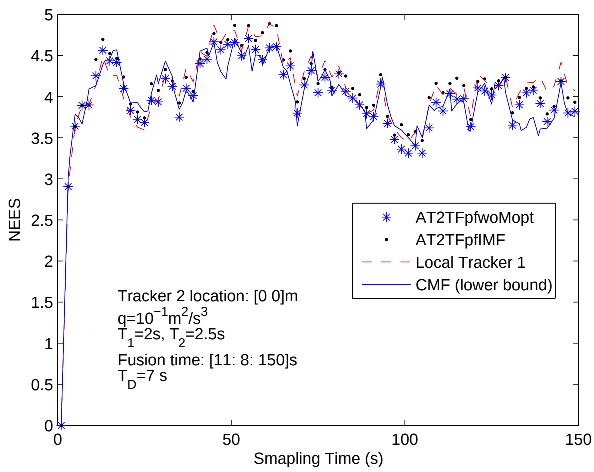

Figure 5: Consistency Test: Setting 2

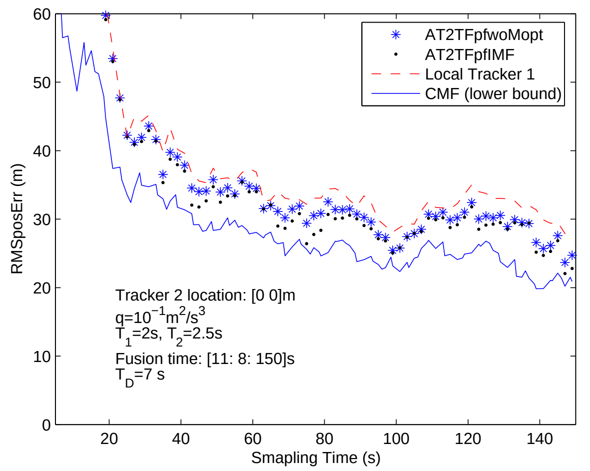

Figure 6: RMS position errors: Setting 2

Figs. 5–6, show that with little or no geometric diversity between the two sensors, AT2TFpfIMF is still consistent and its accuracy is actually better than that of AT2TFpfwoMopt. This is because AT2TFpfIMF belongs to fusion configuration with memory, which utilizes the track estimates from the previous fusion time. Compared to AT2TFpfwoMopt with uses only the latest track estimates (since it operates without memory — woM), fusion with memory is more accurate (see [13] for detailed discussions and the optimal algorithm for fusion with memory).

AT2TFpfIMF (Asynchronous Track-to-Track Fusion with Partial Information Feedback using IMF) vs. AT2TFpfAppr (The Approximate Algorithm for Asynchronous T2TF in [11])

Scenario 3: Fusion of two tracks with high process noise intensity and significant geometric diversity:
Tracker 2 location: $[5000, 0]$ m
Smapling intervals: $T_1 = 2$ s, $T_2 = 3$ s
Process noise PSD: $\tilde{q} = 1\ \mathrm{m}^2/\mathrm{s}^3$
Fusion times: $[5 : 4 : 150]$ s (from 4 s, every 5 s.)
Communication delay: $T_D = 2$ s (For the fusion, local track 2 is about 2 s behind the fusion time.)

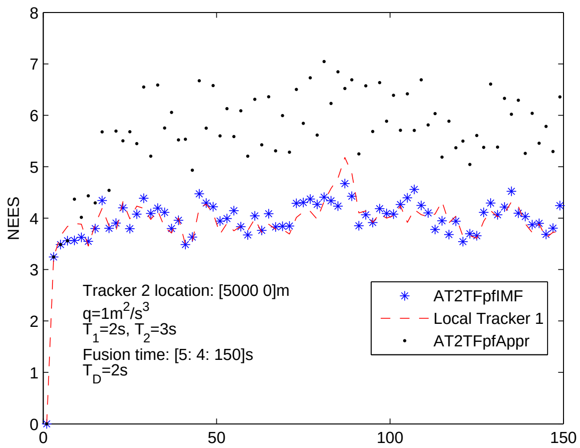

Figure 7: Consistency Test: Scenario 3

Figs. 7–8 compare the performance of AT2TFpfIMF and AT2TFpfAppr against the single tracker 1 without fusion in a scenario with significant geometric diversity and high process noise PSD $\tilde{q} = 1\ \mathrm{m}^2/\mathrm{s}^3$. It can be seen that AT2TFpfIMF is consistent and more accurate than AT2TFpfAppr, while AT2TFpfAppr has a moderate consistency problem when its NEES goes to about 6.7 instead of 4.
Scenario 4: Fusion of two tracks with low process noise intensity and significant geometric diversity:
Here the process noise PSD is $\tilde{q} = 10^{-2}\ \mathrm{m}^2/\mathrm{s}^3$. The rest is as in Scenario 3.

As shown in Fig. 9, the consistency of AT2TFpfAppr becomes worse for a lower process noise PSD, where its

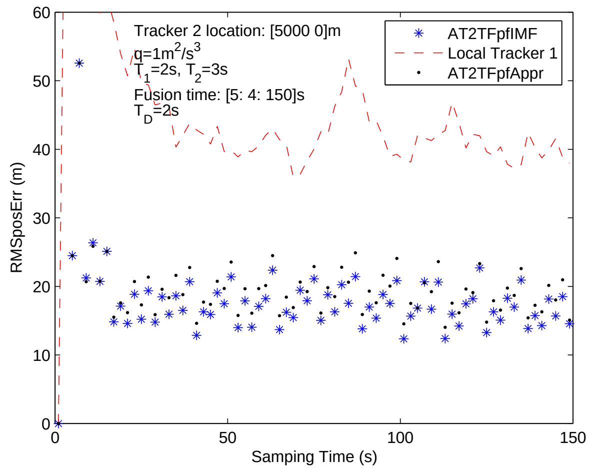

Figure 8: RMS position errors: Scenario 3

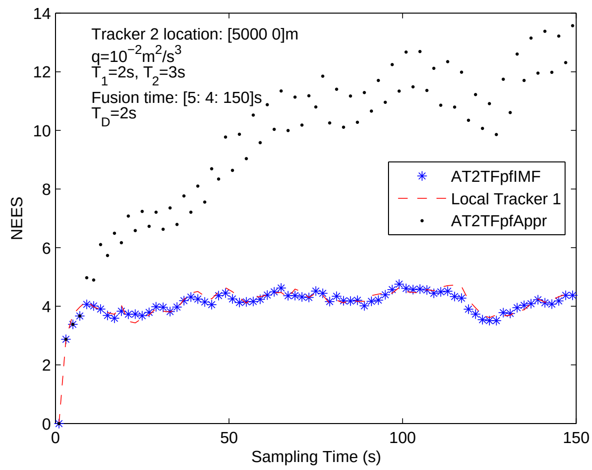

Figure 9: Consistency Test: Scenario 4

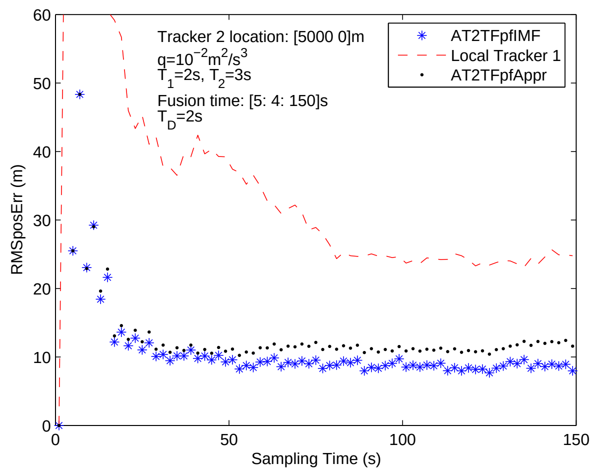

Figure 10: RMS position errors: Scenario 4

NEES grows to 14. This is because that AT2TFpfAppr does not properly account for the crosscorrelations between track 1 and the local track 2 due to the partial information feedback, which plays a more significant role when the process noise intensity is low. Reducing the fusion interval also has a similar effect on the consistency of AT2TFpfAppr which degrades this performance of the algorithm.

6 Conclusions

Three algorithms for the problem of asynchronous track-to-track fusion with delayed tracks and partial information feedback (AT2TFpf) were discussed. Two of them, the optimal AT2TFpf without memory (denoted as AT2TFpfwoMopt) and the information matrix fusion for AT2TFpf (AT2TFpfIMF), have been generalized from their synchronous counterparts. The third algorithm, denoted as AT2TFpfAppr, is an approximate algorithm proposed in [11], based on the algorithm for the fusion of out-of-sequence measurement (OOSM) from [4].

Simulation results show that the heuristic AT2TFpfIMF is remarkably robust. It shows good consistency over the practical range of system process noises and has tracking accuracy as good as AT2TFwoMpfopt when the geometric diversity of the tracks is significant. For the fusion of tracks with little geometric diversity, AT2TFpfIMF is somewhat more accurate than AT2TFwoMpfopt, because it is a fuser with memory. Unlike AT2TFpfIMF and AT2TFwoMpfopt, AT2TFpfAppr is shown to have consistency problems, because it does not properly account for the crosscorrelations due to the information feedback. This leads to degradation in tracking accuracy. In view of AT2TFpfIMF's superior performance and its simple implementation, it is an appealing candidate for practical applications.

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